Variation

This lesson plan is designed to give an overview of Direct Variation (proportionality) and Inverse Variation (inverse proportionality) in line with the Grade 11 Mathematics syllabus in Zambia.
It is suggested that students not be encouraged to take notes, rather that they listen, ask questions, and gain an understanding of the ideas. Further lessons will be necessary to work through details and to give the students vital practice in solving problems.
Equipment: Chalkboard and Chalk.
Duration: About 1 hour.

Introduction

We will be looking at the relationships between things that can be measured, or between things that can be written in equations. We are interested in the idea that in a given type of object or situation, changing one thing can cause a change to another. Let's do a simple example without numbers first.

Call on the tallest student, shortest student, and one in the middle. Draw two axes for a graph on the board – label them arm length on the horizontal, and height on the vertical. Inform the students that we could measure the arm length of the tallest student and his height, and plot that position on the graph – put a representative cross near the top right, pointing out that he has long arms and is also quite tall. Now do the same for the shortest student, putting a cross lower and to the left of the first point, again identifying that this student has rather short arms, and also a shorter body. Repeat for the middle student putting a cross between the first two – suggesting that both arm length and height come between the other two students.

Don't actually measure these values as they are unlikely to produce a proper straight line and we don't want to get into a discussion on experimental error n this maths lesson. Put in the line joining the dots and extend it through the origin.

Explain to the students that the graph indicates that students having greater arm length have greater height. Because it is a straight line, we can say that height varies directly with, or is directly proportional to arm length. Each student only has one length of arm, and one height, but when looking at different students, the relationship between height and arm length can be described this way. There are two ways of writing this relationship in mathematics:

1) $H \alpha L$
2) $H=kL$ (where k is a constant)

Direct Variation

We will now look at some examples which use real numbers so that we can look at the idea of direct variation in more detail.

Maheu Part I

Who likes Maheu (a popular soft drink in Zambia)? I do, so let's draw a table to see how many Maheu I can afford. Let's assume that one Maheu costs 2000 Kwacha. (Write this on the board).

Now create a table – filling in the greyed out values with input from the class, treating each column as a separate example, and getting them to identify the correct ratio for the bottom row.

Money in my pocket (M)	2000 K	4000 K	6000 K	8000 K
No. Maheu I can afford (N)	1	2	3	4
M/N	2000	2000	2000	2000

Plot these values on the graph, and add the trend line through the origin. Point out the general feature that the more money I have, the more Maheus I can buy.

However, there is more to it than that – In every case, the ratio of money to maheus is the same. Also, if we double the amount of money I have, the number of maheus I can buy doubles. Check this with the students for 2000K to 4000K, and 4000K to 8000K. See if there are any questions.

Minibus Part I

Leave the previous table and graph on the board if there is space for another. If not, get the students to remember the shape of it.

Has everyone travelled on a minibus? I have, and recently had to make a number of quite long journeys. Let me tell you about them.

In every case, the minibus was able to travel at a constant 80 km/h. (Put that on the board).

However, each journey was a different length. Do the same thing with this table, working through each greyed out example with the students.

Distance Travelled (D)	80 km	160 km	240 km	320 km
Time Spent in Minibus (T)	1 hr	2 hr	3 hr	4 hr
D/T	80	80	80	80

On a new graph, (Time versus Distance) plot the new data with the trend line. Identify the same features – the greater the distance, the longer the journey took. Every time the distance doubled (80 to 160, 160 to 320) the time spent also doubled. Also, the ratio of distance to time was constant.

Summary

Look at the two examples – point out that we could write $N \alpha M$, or $N=kM$ in the first case. Can they work out the value of k? (=1/2000 as found by rearranging the equation.)

In the second case, the relationship could be described by writing $T \alpha D$, or $T=cD$. Get the students to find c for this case.

This is Direct Variaton

Inverse Variation

In the work we just did, we looked at cases where making one number bigger caused another number to get bigger. We're now going to look at situations where making one number bigger makes another number smaller.

Maheu Part II

Clear the board.

We're now going to look at Maheu again – only this time, I've always got the same amount of money in my pocket every day – 8000 Kwacha (put this on the board.) However, depending on which shop

I go to, the price of one Maheu can be quite different. Of course, this affects how many I can afford.

Here is the table of data to work through with the students. Note that this time, we're putting the product of the first two values on the bottom row, not the ratio. Before going through the grey data, see if any students can guess what the graph is going to look like. Don't confirm or deny their answer yet – let the data do its work! At the time of writing, 2000 K is a fairly standard value for one Maheu – this makes 8000 K a ridiculous price, but it's OK to just have a laugh about this with the students. The ridiculous tends to stay in the memory!

Cost of Maheu (C)	1000 K	2000 K	4000 K	8000 K
No. Maheu I can afford (N)	8	4	2	1
CxN	8000	8000	8000	8000

Plot the graph – it may be good to put the first and last points on first, then the middle ones. This will help highlight the general trend being from top left to bottom right, but the shape not being a straight line.
Point out that the product of the two variables is always the same. That the more each Maheu costs, the fewer I can afford, and that every time the cost doubles, the number I can afford halves.
Another way of saying this is that every time the cost is multiplied by two, the number I can afford is divided by two.
This is what is meant by inverse variation, or inverse proportionality.

Minibus Part II

Now we revisit minibus journeys – this time looking at a time when I had to travel 320 km, so went to speak to several different drivers to see how fast his vehicle could go. (put 320km on the board)

As in the previous example, the bottom row is a product, not a ratio.

Speed of minibus (S)	20 km/h	40 km/h	64 km/h	80 km/h
Time spent in minibus (T)	16 hrs	8 hrs	5 hrs	4 hrs
SxT	320	320	320	320

Reinforce the idea that the product of S and T is a constant and that every time the speed is multiplied by 2, the time of the journey is divided by 2. In general, the bigger the distance, the smaller the amount of time taken.

Summary

Look at the two examples – point out that we could write $N \alpha 1/C$, or $N=p/C$ in the first case. Can they work out the value of p? (=2000 as found by rearranging the equation.)

In the second case, the relationship could be described by writing $T \alpha 1/S$, or $T=q/S$. Get the students to find q for this case.

This is Inverse Variation.