Kinematics and making predictive calculations

Updated May 3, 2009

Materials:

- meter stick or tape measure
- toy cars that roll with very little friction (note: do not substitute a rolling spherical object, since the rotational kinetic energy would fundamentally change the analysis)
- flat smooth board or other surface for ramp, 70-100 cm long or so.
- cardboard (not corrugated) or thick paper to round off the bottom of ramp (see figures)
- sticky tape
- desk or table. Must be level.
- cloth, rag, towel, or old shirt to use as a “landing pad”
- string and small heavy object for a plumb bob
- rock or other object to prop up the ramp
- spirit level. If you don’t have one, a wide flat-bottomed cooking pan with water can be used (see below)

Suitable for: 3-30 students. Pupils should be familiar with these three equations:

In time t, a freely falling body will fall a distance $h = gt^2/2$.

Gravitational potential energy = mgh.

Kinetic energy = $mv^2/2$.

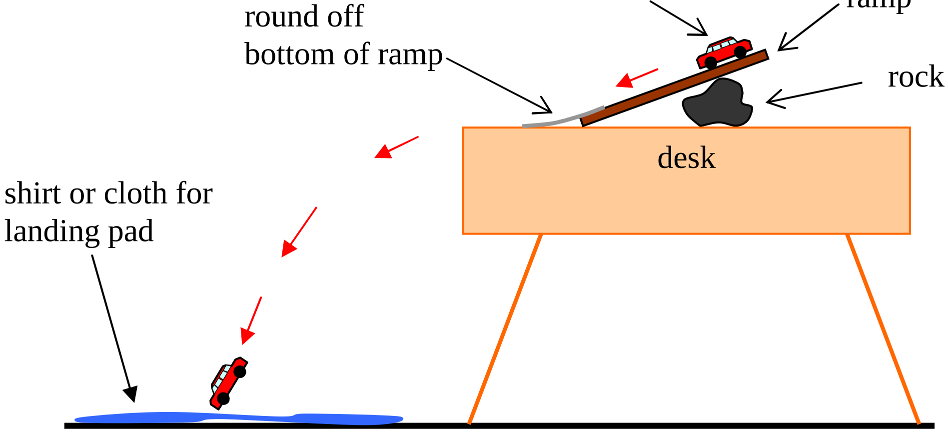
Estimated time: 2 hrs. The activity nicely breaks up into 1hr on intro/review/theory and 1hr on experiment/analysis, so it could be split between two lessons of one hour each.

Topics covered: equations of motion, conservation of energy, friction, graphing data, changing the subject of an equation

I. Apparatus

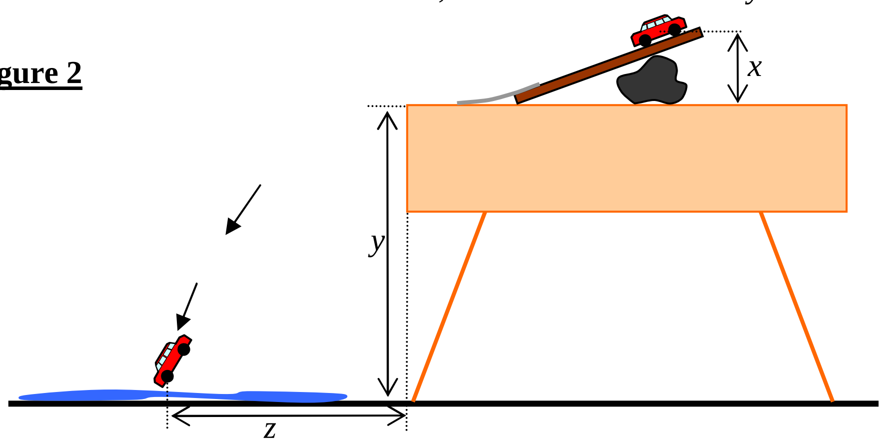
Before attempting the lesson with students, prepare the following apparatus to make sure you have all the necessary materials:

Figure 1



The basic plan is that we will release the car on the ramp; the car will roll down the ramp and off the desk, flying through the air and hitting the floor. Based on measurements of the ramp and of the desk, we will try to calculate where the car will land using some equations from the physics syllabus. In terms of the illustration below, we will measure x and y and thereby calculate z:

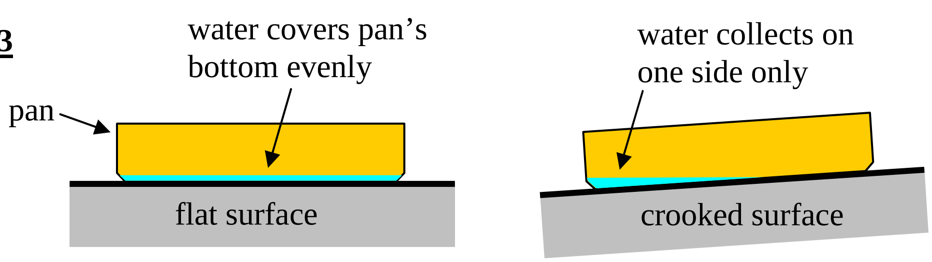
Figure 2



x = starting height of car above surface of desk
y = height of surface of desk above the floor
z = distance car travels horizontally past the edge of the desk

The floor and surface of the desk both need to be precisely horizontal; if either surface is crooked then our calculations later on will be very inaccurate. If you do not have a spirit level to test the surfaces, you can make one with a wide flat-bottomed cooking pan: place a small amount of water in the pan, and observe whether or not the water covers the bottom evenly:

Figure 3

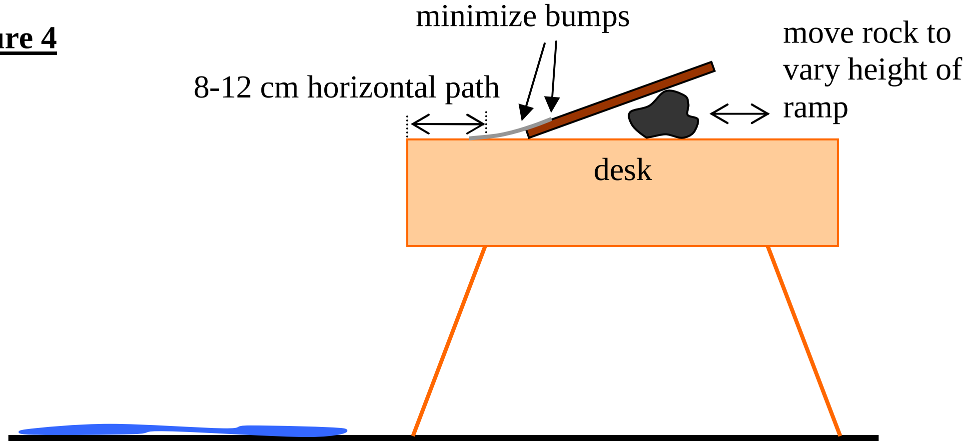


The cardboard or thick paper should be secured with tape to the bottom of the ramp so the car goes smoothly from the ramp to the desk. Try to minimize any bumps the car would feel as it rolls, since bumps will cause the car to travel slower than the speed we calculate. Also make sure that the ramp does not slip or move at all as the car rolls down it – even a slight motion will invalidate our calculation.

We will want to vary the angle of the ramp during the experiment. This angle can be adjusted by moving the rock underneath the ramp. I found it useful to tape the bottom of the ramp to the desk so it did not move as I moved the rock.

It is also crucial that the car be traveling exactly horizontal when it leaves the desk. This means there should be about 8-12 cm of horizontal "road" from the bottom of the ramp to the end of the desk:

Figure 4



The shirt or cloth is placed in the region where the car lands to soften the impact and prevent the car from breaking.

II. Introduce lesson to students

Begin the lesson by having the students gather around the apparatus. Roll the car down the ramp once or twice to illustrate the car's trajectory. Show that the angle of the ramp can be varied. Explain to the students that we will use the equations of motion to understand why the car travels the distance it does.

Point out that calculations of this sort are important in many real-world situations. You want to calculate performance *before* investing time and money in constructing the real thing. For example, how thick should you make the beams of a building in order to support the weight of the roof? How much electricity will a mobile phone tower need in order to provide reception to the surrounding neighborhood? In all of these situations, we can use physics and mathematics to calculate how the system will behave before we invest the time and money in building it.

III. Theory

Draw Figure 2 on the chalkboard so everyone can remember the definitions of x, y, and z.

Tell the students that we will now work through several steps to calculate a formula for z.

Ask the students "What is the change in the potential energy of the car between the top and bottom of the ramp?"
Answer: gravitational potential energy = *mgh*

Ask the students what *g* means.
Answer: 9.8 m/s^2

Give the students a multiple-choice question:
"On the moon, *g* is
   A) more than 9.8 m/s^2
   B) exactly 9.8 m/s^2
   C) less than 9.8 m/s^2 but more than zero
   D) zero"
Have every student write down their answer.
Answer: C) There is gravity on the moon, but it is weaker than here on Earth.

Ask: "What is the formula for the kinetic energy of the car at the bottom of the ramp?"
Answer: kinetic energy = $mv^2/2$

Ask "What is the kinetic energy of the car at the top of the ramp?"
Answer: zero, since the speed *v* is zero there.

Ask the students to write an equation which expresses the conservation of energy and which uses these formulae we have just discussed.
Answer: $mgx = \frac{1}{2}mv^2$

Have the students change the subject of the equation to *v*:
Answer: $v = \sqrt{2gx}$

Next, define the time *t* to be the number of seconds between the moment the car leaves the ramp and the moment the car hits the ground. Ask the students for the formula which relates the distance *y* to the time *t*. (Perhaps write this question on the board for clarity.)
Answer: $y = \frac{1}{2}gt^2$
Have the students change the subject of this equation to *t*:
Answer: $t = \sqrt{2y/g}$

Next, tell the students that the horizontal speed of the car does not change while it is in the air. Ask the students: "Therefore, what equation can we write that relates *z*, *t*, and *v*?"
Answer: $z = vt$. Point out that this equation means *distance* = *speed* × *time*.

Ask if this is a useful formula for predicting *z* based on numbers we can easily measure.
Answer: the formula is not very useful the way it is written, because *v* and *t* are both hard to

measure. Ask: “Which quantities are easier to measure?” Answer: x and y are easier to measure because they are distances.

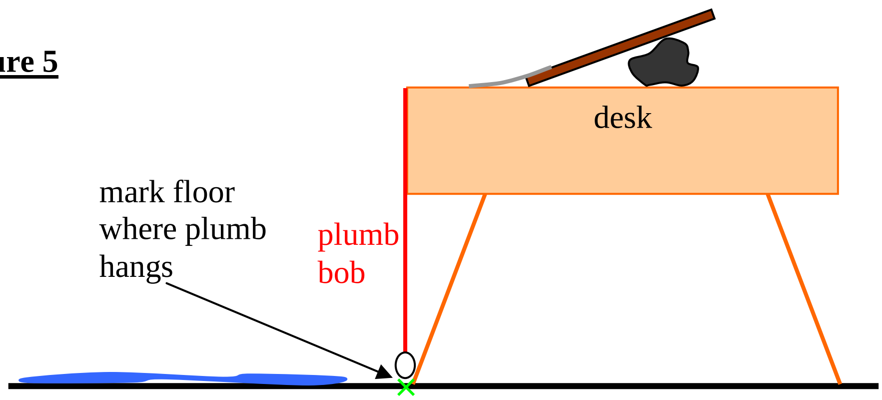
Finally, have the class find an equation which tells us z using only numbers which are easy to measure.
Answer: combine our three equations $v = \sqrt{2gx}$, $t = \sqrt{2y/g}$, and $z = vt$ to obtain $z = 2\sqrt{xy}$

Emphasize that if we measure x and y, then this formula tells us what z should be, even if we do not measure the weight of the car or the length of the ramp. We also do not need to measure speed or time at all, which is good because t is only about a second and it would be hard to measure precisely.

IV. Experiment

First we need to mark the point on the floor which is directly underneath the end of the desk. We do this using a “plumb bob”, which consists of a string with a heavy object attached at the end (a screw, bolt, or small stone works well). Hang the string from the edge of the desk at the point where the car would begin its flight. The weight will cause the string to hang exactly vertically. The weight should hang just above the floor without touching it. Have a student mark the point on the floor directly underneath the weight (chalk or tape works well for marking this point). When we measure the distance z we will measure from this mark.

<u>Figure 5</u>

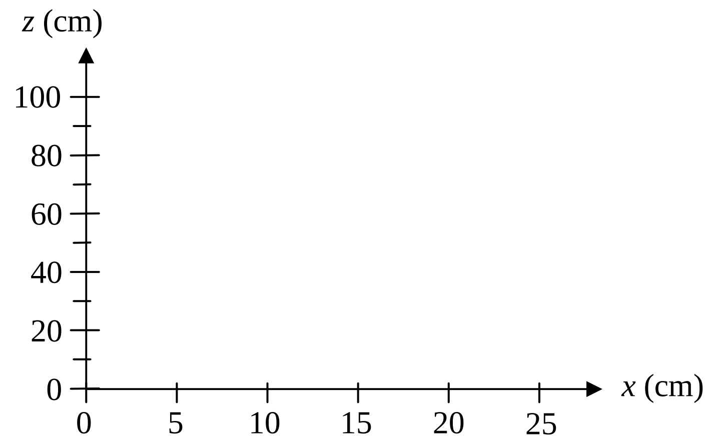


The plumb bob can now be removed.

Have a student measure y, the distance from the mark on the floor to the edge of the desk. Record this number on the chalkboard.

Draw the following set of axes on the chalkboard:

Figure 6



We will plot our data and our calculations on these axes as we take measurements.

Hold the car at the start position atop the ramp. Have a student measure *x*, the height of the middle of the car above the surface of the desk when the car is in the start position. Now have each student in the class use the formula $z = 2\sqrt{xy}$ to calculate how far we predict the car will travel. Have a student plot this point on the chalkboard axes of Figure 6 with a small circle.

Now put the car back on the ramp at the start position and let it go. Do not push the car at all – just release it gently so that gravity is the only force pushing it. A student should be watching the cloth on the floor when the car is released to identify exactly where the car lands – the student should hold his or her finger there to temporarily record the location. Now another student should measure the distance between this point and the mark from the plumb bob (i.e., measure *z*). Have a student record the data point on the chalkboard axes with an 'X'.

Send the car down the ramp a few more times, measuring *z* after each trial and recording each value on the chalkboard graph.

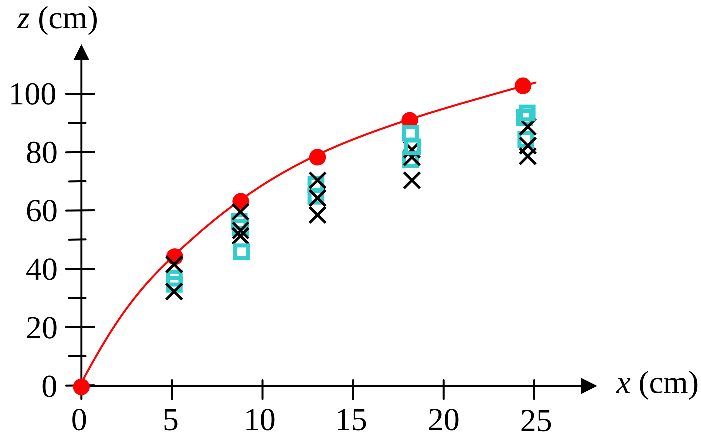
Next, ask the students "If we used a heavier toy car, do you think it would travel further or not?" Have each student write down their prediction. Ask: "Does our formula $z = 2\sqrt{xy}$ predict that a heavy car will go further, less far, or the same distance?" Answer: the mass of the car does not appear in the formula, so the calculated *z* is the same for both cars. Let the students change their written predictions if necessary after revealing this answer.

Now repeat the experiment several times with a "heavy car", made by attaching a bolt, small stone, or other weight to the car with tape. Instead of marking the data on the graph with an 'X', use a square.

Now change the angle of the ramp. Measure the new value of *x*, and again have the class calculate the predicted *z* using $z = 2\sqrt{xy}$. Have a student record this point on the graph with a circle. Roll both the light car and the heavy car down the ramp a few times, recording the *z* value on the graph after each trial as before. Always record the data for the light car with an 'X' and record the data for the heavy car with a square

Repeat these last steps for at least 5 different ramp angles. The graph should look like this:

Figure 7



V. Analysis

Discuss why the measurements were not exactly the same on each trial. Was there any systematic difference between the data and the prediction? What are the possible sources of this systematic error? (Friction or bumps on the ramp, friction in the air, motion of the ramp itself if it is not sufficiently rigid, ambiguity in measuring the initial height of the car or the landing point, rotational energy of the car and its wheels.)

Have the students compare the data with their written predictions about whether the weight of the car would affect *z*.

Ask the class: "Although the calculation consistently over-predicted *z*, what are some features of the data which the calculation *does* correctly predict?"
Answers:

- The distance traveled does not depend strongly on the weight of the car
- The curve is not a straight line: as you increase the ramp height *x*, the distance traveled *z* first increases rapidly but later increases only slowly.

Optional challenge question: "If you performed this same experiment on the Moon, using a desk of exactly the same height (i.e., the same value of *y*), how would the graph turn out?"
Answer: Since the strength of gravity *g* does not appear in our formula $z = 2\sqrt{xy}$, the predictions would be exactly the same! Though the car would roll slower since gravity was weaker, the car would also fall more slowly, and the two effects cancel out in such a way that the distance *z* does not depend on the strength of gravity! Also, the data might lie closer to the predictions because there is no air resistance on the moon.

Finally, ask the for 1-3 volunteers from the class to explain in their own words: "What was the point of this activity? What did you learn?"