Parallax: How we measure the distances to stars

Materials required:

- Tape measure
- A flat rigid piece of wood or Styrofoam, at least 30cm long and at least 15cm wide
- 3 thumbtacks
- A round vertical pole, less than 10cm diameter. Anything from a structural column to a chemistry stand would work.
- A second pole like the one above, 8-12m away from the first.
- 2m of sturdy string
- One picture each of a star, of the earth, of a galaxy, and of the sun. Each can be drawn by hand on a separate sheet of paper.
- Calculator
- Chair or desk

Suitable for: 3-30 students

Estimated time: 2 hours

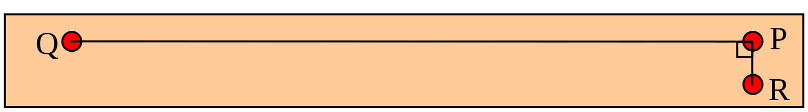
Topics discussed: parallel lines, similar triangles, trigonometry, changing the subject of an equation, measuring distances, motion of the earth around the sun, astronomical objects.

I. Preparation

Before any students arrive, plan out which space you will use for the measurement and where you will place the galaxy picture: see Figure 4.

Also, draw two *perpendicular* lines on the flat piece of wood or Styrofoam as shown below. In the rest of this document I will call this object the 'board.' Place one thumbtack at the intersection of the lines. Also place one thumbtack on each of the lines. Write "P", "Q", and "R" on the board to label the three pins in the order shown here:

Figure 1:



board

P-Q-R should form a right angle.

To make sure you understand all of the steps, it is best if you carry out the experiment in Section V on your own before giving the lesson to students.

II. Introduction

When the students arrive, tell them that the nearest star after the sun, Proxima Centauri, is 40,000,000,000 km (4 with 13 zeros!) away from the Earth. This star was discovered by a South African astronomer, Robert Innes. This huge number is one example of the enormously large or enormously small numbers we hear about in science. How could we possibly measure a distance this large? There is no way to do it directly with a ruler! Today we will work through one method which is actually used by astronomers to perform this kind of measurement. The activity shows how the combination of careful measurement with clever mathematics can be used to determine remarkable facts about the universe. We will make use of the idea of similar triangles and trigonometry which have been discussed on the mathematics syllabus.

III. Defining parallax

First, draw the following image of a view out the **side** window of a minibus:

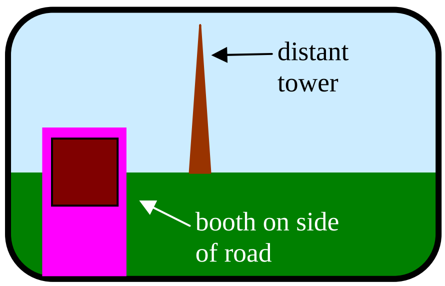


Figure 2:

You see a booth for a vendor selling top-up cards by the side of the road, and far off in the distance, you see a tower of some sort. A moment later, your view out the window might look instead like this:

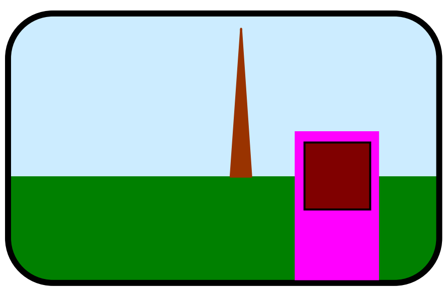
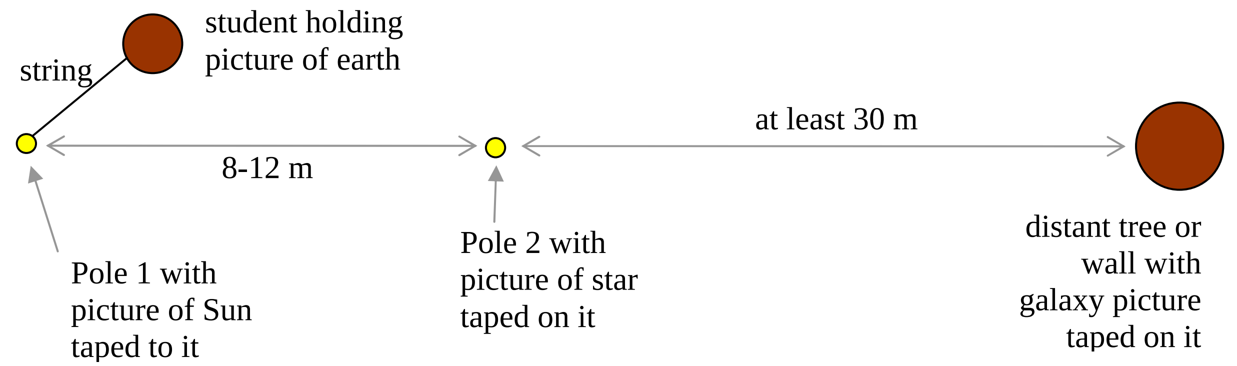


Figure 3:

The tower has moved slightly compared to the frame of the window, but the booth has moved quite a bit. Stated another way, the booth appears to move past the window very quickly compared to the tower. In general, nearby objects seem to go past your view faster than objects far away. Also, note that at first (Figure 2) the booth was to the *left* of the tower, but later on (Figure 3) the booth is now to the *right* of the tower. This effect is called "parallax." Since the apparent motion of objects is related to their distance from you (greater motion for nearby objects, less motion for distant objects), then we can use a measurement of the apparent motion to determine how far away an object is.

Arrange the following setup. You should definitely plan out the placement of the poles and galaxy before the students arrive for the activity.

Figure 4: (not to scale!)



For "Pole 1" and "Pole 2", you could use a structural column, a signpost, a chemistry stand, or any other rigid vertical object.

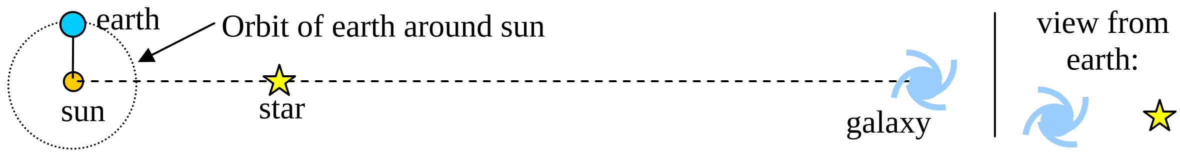
Figure 5: These structural posts worked great for Pole 1 and Pole 2

The two poles and the galaxy should all lie on a straight line; you should verify this by standing behind Pole 1 and looking towards these three objects. The height of the galaxy picture above the ground should be set so that it can be lined up with Pin Q and Pin R in Section V.

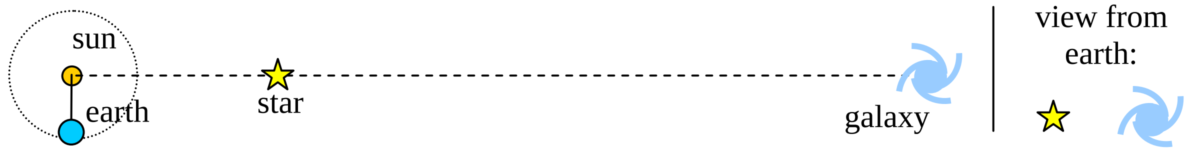
One end of the string is tied loosely around Pole 1. The student holds the other end of the string, and he/she holds the picture of the Earth; perhaps you could tape the picture to the student's shirt. The student should be able to walk in a circular "orbit" around Pole 1 (i.e. the Sun), ensuring that he/she is always the same distance from the Sun by keeping the string taught. The sun-earth distance should be roughly 1 m, but the precise distance is not important.

Have the student walk around the sun with the string taught in an "orbit." Have them observe that depending on their orbital position, the near star could be either to the left or right of the distant star:

Figure 6: “Bird’s eye” view: (i.e., north points out of the page, south points into the page)



Above, the star appears to the *right* of the galaxy from earth. But 6 months later, when the Earth has completed ½ of an orbit, the star appears to the *left* of the galaxy, as shown below:

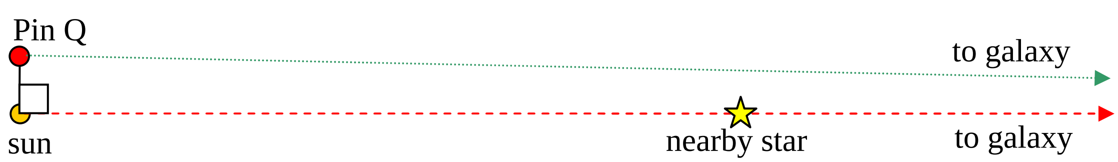


Emphasize that this is the *same* parallax effect we found looking out the window of a minibus. Tell the class that we will use a measurement of this effect to find the distance to the nearby star. Give other students a change to hold the string and act as the earth so they can see the parallax effect.

IV. Geometrical analysis

Bring the students back to the classroom to describe how we’ll make the measurement. Draw the figure below on the chalkboard:

Figure 7:

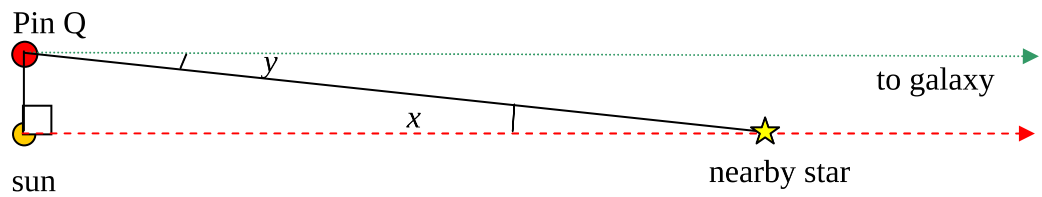


“Pin Q” represents the thumbtack you have placed in the wood or styrofoam. Explain that Pin Q is basically the position of the earth. Therefore, this picture represents a time of year when the earth-sun-star angle happens to be 90°.

We first need to argue that if the galaxy is far enough away from the earth and sun, then the Q-galaxy line is nearly parallel to the sun-galaxy line. Put another way, in the above figure, the green (dotted) and red (dashed) lines are *nearly* parallel. These two lines are not *exactly* parallel, because they will eventually cross at the galaxy (far to the right off the edge of the illustration), but the lines are *nearly* parallel if we are only looking in the region of the earth and the star.

Next, add the line and labels shown below to the chalkboard figure:

Figure 8:



x = Q-star-sun angle (basically the same as the *earth*-star-sun angle)
y = galaxy-Q-star angle (basically the same as the galaxy-*earth*-star angle)

Give the students the following multiple-choice question:

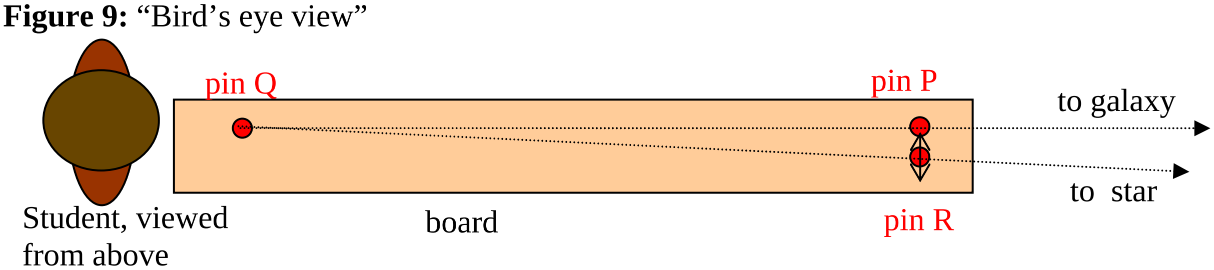
A) $x = 2\,y$
B) $x = y/2$
C) $x = 180^{\circ} - y$
D) $x = y$
E) I don't know

Give the students a minute to discuss this question with their classmates. See if any students can make an argument for why they believe one of the choices is the correct one. Have each student write down their answer. Now reveal the correct answer: D. The reason is that x and y are *alternate angles* associated with the "parallel" dotted and dashed lines in the figure, and there is a theorem in geometry that alternate angles are equal.

Also argue that the Q-sun-star angle is 90 degrees at roughly the time of year that the angles x and y reach their *maximum* value.

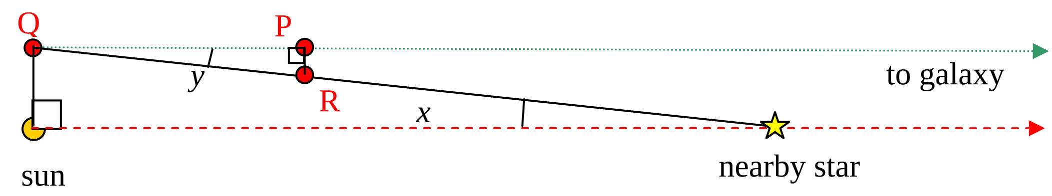
Now show the students the board with the lines and thumbtacks. Explain that we will line up pins Q and P with the galaxy, and then we will move pin R along its line until pins Q and R are lined up with the nearby star:

Figure 9: "Bird's eye view"



Now, add points P and R to the chalkboard diagram as follows:

Figure 10:



Point out that Q and P form a line pointing to the galaxy, while Q and R form a line pointing to the nearby star.

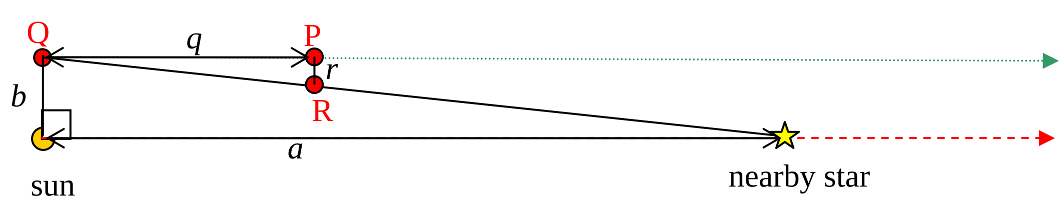
Now give give the students the following multiple-choice question: "The P-Q-R triangle and the Q-sun-star triangle are:

A) identical
B) congruent
C) similar
D) obtuse
E) I don't know"

Give the students a minute to discuss this question with their classmates. See if any students can make an argument for why they believe one of the choices is the correct one. Have each student write down their answer. Now reveal the correct answer: C: the triangles are *similar*. The reason is that the two triangles have the same angles (because $x = y$) even though they have different side lengths. Remind the students of the definition of similar, congruent, and obtuse.

Now, suppose we define the following distances:

Figure 11:



a = distance from sun to nearby star
b = distance from Q to the sun
q = distance from Q to P
r = distance from P to R

The figure above is not to scale, because the earth-sun distance b will really be much larger than q or r. It would be difficult to make the drawing to scale.

Now ask the class: "What equation can we write to relate the distances a, b, q, and r?" There are several possible correct answers, including $\frac{a}{b} = \frac{q}{r}$, $\frac{a}{q} = \frac{b}{r}$, $\frac{b}{a} = \frac{r}{q}$, and $\frac{q}{a} = \frac{r}{b}$. These answers all follow from our earlier finding that the P-Q-R triangle and the Q-sun-star triangle are similar.

Now have your students change the subject of the equation to *a*. Regardless of which equation you use in the previous step, the correct answer always turns out to be $a = \frac{qb}{r}$.

Next, we will obtain the same result in a different manner using the tangent function from trigonometry. Ask your class: "Looking only at the Q-sun-star triangle, what is tan(*x*)?" Correct answer: $\tan(x) = b/a$. Now ask your class: "Looking only at the Q-P-R triangle, what is tan(*y*)?" Correct answer: $\tan(y) = r/q$.

Next, point out that since $x = y$, we can write

$\frac{b}{a} = \tan(x) = \tan(y) = \frac{r}{q}$ and so the first fraction must equal the last fraction: $\frac{b}{a} = \frac{r}{q}$.

Now have your students change the subject of the equation to *a* again. Correct answer: $a = \frac{qb}{r}$, the same answer we obtained before! Both the "tangent method" and the "similar triangles method" give the same result.

V. Experiment

Now state to the class that we have all the pieces we need to measure the distance from the sun to the star:

- First we find the orbital position that gives the maximum apparent separation between the star and the galaxy (i.e. the maximum angle *y*).
- Next we rotate the board so pins Q and P line up with the galaxy.
- Then we move pin R so that Q and R line up with the star.
- We measure the distances *q* and *r*. We also need to know the distance *b*.
- Finally, we apply the formula $a=qb/r$ to calculate *a*.

When astronomers actually use this parallax method to measure the distances to stars, they need to know the earth-sun distance (*b*). It is complicated to explain how this distance is known; if students are curious, you can just tell them it involves precise measurements of lunar eclipses and of the phases of the moon. For this lesson, we will just measure *b* directly.

Now return to the outdoor setup from Figure 4. Set the board on a desk or chair so that it is horizontal, with the thumbtacks sticking up. Tie one end of the string to Pole 1 and tie the other end to Pin Q so that there is roughly 1m between Pole 1 and Pin Q.

Next, have the students find the orbital position that gives the maximum parallax. In other words, move the board around Pole 1, keeping the string taught, until the apparent separation between the star and the galaxy is the maximum possible. (Move the desk or chair as needed). This maximization does not need to be very precise.

Now have the students rotate the board (keeping it horizontal) until Pin P, Pin Q, and the galaxy all are lined up. Do this by crouching down so that your eye is just behind pin Q. You should be able to see Pin Q and Pin P directly in front of the galaxy picture and overlapping each other at the same time. This alignment is easiest if the height of the galaxy picture above the ground has been set appropriately, as I mentioned just after Figure 5. The string should remain taught. Check the alignment to make sure the students have done it correctly.

Next, have the students move Pin R along the line you have drawn on the board. Position Pin R so that Pin Q, Pin R, and Pole 2 (i.e. the nearby star) all are lined up. Do this by crouching down so that your eye is just behind pin Q. You should be able to see Pin Q and Pin R directly in front of Pole 2 and overlapping each other at the same time. Check the alignment to make sure the students have done it correctly. Be very careful not to bump or rotate the board while Pin R is being placed, since this will destroy the Q-P-galaxy alignment from the previous step. It the board is bumped or rotated by even a slight amount, go back and repeat the Q-P-galaxy alignment step.

Once Pin R is properly fixed in place, then it is again safe to move the board. Have the students measure the distances q and r. Also have the students measure the distance b from Pin Q to Pole 1.

Have the students plug the measurements into the formula $a=qb/r$ to calculate a.

Finally, use the tape measure to actually measure a directly. You should get a value which is relatively close (within 30cm) of the value you calculated. Point out that you could not actually do this direct measurement of the distance from the sun to another star. For our simulation here, however, we do this direct measurement to check that it agrees with our calculation from the parallax method.

Have the class discuss possible reasons for disagreement between the calculated and measured values for a.

Mention that the method can be extended to the case where we are not lucky enough to have a distant galaxy which is in a straight line (collinear) with the sun and the nearby star.

VI. Recap

Have the students summarize the procedure (the bulleted points at the start of Section V) in their own words.

Ask the students what they learned. What were the key ideas we learned and used in this lesson?

Emphasize:

- A clever measurement can be combined with mathematics to learn something about the world (such as the distance to a star) which is much bigger than human scale.
- Ideas from mathematics – such as similar triangles, the tangent function, alternate angles, and changing the subject of an equation – can be useful for something!